Why the sky is blue (on a clear day with an unpolluted Earth atmosphere)

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Abstract

Light from the sun passing through Earths unpolluted atmosphere is primarily scattered conform the Rayleigh law, which states the scattering is a function of λ^{-4}. Therefor the violet/blue end of the spectrum is scattered more. The human eye is more sensitive to blue than to violet, so the sky is perceived to be blue.

Contents

Introduction

Why the Sky is Blue, a Poem by John Ciardi
I don't suppose you happen to know
Why the sky is blue? It's because the snow
takes out the white. That leaves it clean
For the trees and grass to take out the green.
Then pears and bananas start to mellow,
and bit by bit they take out the yellow.
The sunsets, of course, take out the red
and pour it into the ocean bed
or behind the mountains in the west.
You take all that out and the rest
couldn't be anything else but blue.
Look for yourself. You can see it's true.

The question "why is the sky blue" seems rather trivial. But, as is often the case, appearances are deceiving. The answer to this has eluded philosophers and physicists for centuries, and the first decent approximation for an answer was made as late as the very end of the 19th century. And even although the underlying principles are known today the formulas still do not provide 100% accurate answers... In this article, written for the 2004 course "Keerpunten in de Natuurwetenschappen" given by the Institute for Interdisciplinary Studies of the Universiteit van Amsterdam I will try to give a decent answer by examining the various things that happen to light before they reach our eyes. I will start by looking at the source of light, our sun itself, and end inside the human eye.

1 The light from the sun

The sun emits electromagnetic radiation over a wide range of wavelengths. To approximate the emitted light intensity as a function of the wavelength mathematically assume the sun qualifies as a 'black body'. It is then given by the Planck equation:

$$I_T(\lambda)\Delta\lambda = \frac{2\pi hc^2\Delta\lambda}{\lambda^5 e^{hc/\lambda kT-1}} \tag{1}$$

Where $I_T(\lambda)\Delta\lambda$ is the emitted energy per unit area of blackbody per unit time within a wavelength interval $\Delta\lambda$, measured at absolute temperature T and wavelength λ and k is Boltzmann's constant. Graphs for several temperatures have been plotted in figures 1 and 3. The latter also shows the real measured curve. The peak wavelength is given by Wien's displacement law:

$$\lambda_{max} = \frac{2.898x10^{-3}mK}{T} \tag{2}$$

Taking 5800K as the surface temperature of the sun, the peak wavelength is therefor at about 500 nm. This is in the blue-green part of what is called 'the visible spectrum', or the range of frequencies that human eyes can see. The fact that the suns intensity is peaked at blue-green does however not mean that it is perceived as blue-green. The visual spectrum covers the range from about 380 nm (violet) to about 740 nm (red), and discerns between colors by *adding* the

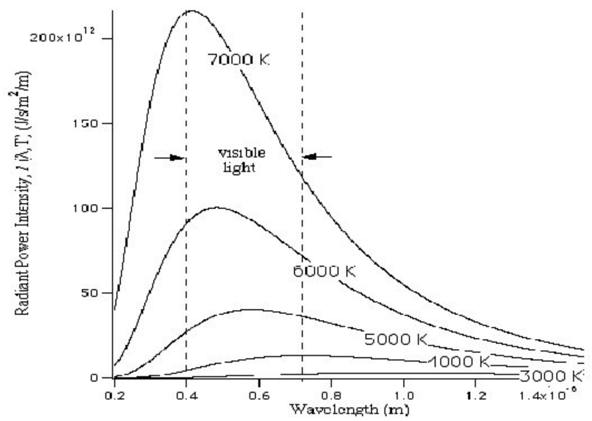

Figure 1: Planck radiation curves for several temperatures. For the sun T=5800K

Gas	Formula	Abundance % by volume
Nitrogen	N_2	78.084%
Oxygen	O_2	20.9476%
Argon	Ar	0.934%
Carbon dioxide	CO_2	0.0314%
Neon	Ne	0.001818%
Helium	He	0.000524%
Methane	CH_4	0.0002%
Krypton	Kr	0.000114%
Hydrogen	H_2	0.00005%
Xenon	Xe	0.0000087%

Figure 2: Composition of Earths atmosphere. Source: CRC Handbook of Chemistry and Physics, 77th Edition

(discriminated) intensity of several wavelengths. This will be explained in more detail later, but for now it is only important to know that the sun sends out light at all visual frequencies.

2 Radiation Interaction with the Atmosphere

Not all of the solar radiation actually reaches an observer on Earth due to interactions with the atmosphere, mainly absorption, scattering, and reflection. Many of these interactions are non-selective meaning they do not depend on the wavelength of the incoming light and interact without discriminating. Some however do favor certain wavelengths; and these are the kind of interactions that may be responsible for the color of the sky.

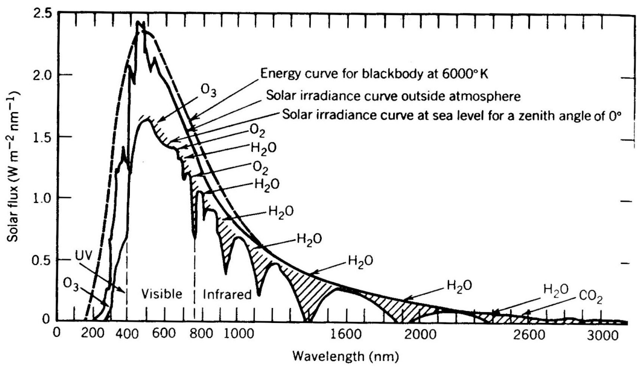

Figure 3: Solar radiation incident at the top of atmosphere, troposphere and surface (Howard and al., 1960)

2.1 Composition of Earths atmosphere

As shown in the table, Earths atmosphere consists mostly of Nitrogen and Oxygen and less than 0.04% trace gases. Not mentioned in the table, because the exact amount is strongly location dependant, is water vapor which varies between 0-7% (the amount present is called the *humidity* of the region). Also found in various amounts are aerosols, water and ice. Aerosols are tiny liquid droplets, such as fog, or tiny solid particles, such as ice crystals, smoke, sea salt crystals, dust, and volcanic emissions, suspended in the air. The typical size of the gaseous molecules is of the order of 1 nm, while aerosols can range in size from 10 to 10000 nm; where the larger are e.g. found in clouds.

2.2 Selective absorption

Absorbtion is the process by which incident radiant energy is 'retained' by a substance. Absorption results in the conversion of radiation into some other form of energy within and according to the nature of the absorbing medium. The absorbing medium itself may emit radiation, but only after an energy conversion. Figure 3 shows that selective absorbtion indeed occurs within earth's atmosphere, but that it mostly does this *outside* the visual spectrum. The gamma rays, X-rays, and ultraviolet radiation less than 200 nm in wavelength are absorbed by oxygen and nitrogen. Most of the radiation with a range of wavelengths from 200 to 300 nm is absorbed by the ozone layer, while the red and infrared regions of the spectrum at wavelengths greater than 700 nm are absorbed to some extent by carbon dioxide, ozone, and water. But within the 400-700 range which can be viewed by humans there is an 'atmospheric window' in which selective absorbtion is at a minimum.

However, many physicists (Newton, Da Vinci, Clausius) throughout history considered it likely that 'atmospheric water' in some form, like vapor or globules, was the cause of the blue sky. The main reason for this is that liquid water (and ice) do in fact absorb light in the red end of the visible spectrum - so when observing light that has passed through several meters of water it appears blue[1].

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[1] which means we hereby have part of the answer to the question 'Why is the ocean blue ?'

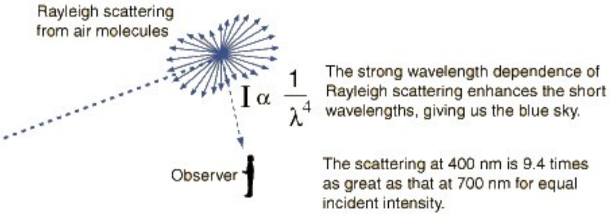

Figure 4: Rayleigh scattering explained

The amount of water vapor in the atmosphere is however too little to cause the blue sky; in fact if all the water in the atmosphere were to be compressed into a liquid the result would be a layer of only 1 cm in thickness - which is nowhere near the required meters.

2.3 Scattering

We say light is scattered if it is emitted in different directions after interacting with a molecule or particle (which we will call the scatterer). Scattered light is radiation from matter excited by an external source (for the atmosphere: the sun). So without this source (at night) there is no scattering. Selective scattering is so named because radiations with shorter wavelengths are selectively scattered much more extensively than those with longer wavelengths. This is in fact the most widely accepted reason the sky is blue...

2.3.1 Rayleigh Scattering

In 1871, Sir William Strut(better known as the Baron of Rayleigh) used dimensional analysis to derive a formula for intensity of scattering by (dipole) particles that are small compared to the wavelength of the incident light:

$$I = I_0 \frac{8\pi^4 \alpha^2 N}{\lambda^4 R^2}(1 + cos^2\theta) \tag{3}$$

Here I is the intensity, α a measurement for the polarizabiliaty, λ the wavelength and N the number of scattering molecules, where it is assumed the atmosphere is an ideal gas so that scattering by N molecules is N times the scattering by one. A derivation of this formula using electrostatic principles is included in the appendix. It was later improved for atmospheric scattering using various correction factors by physicists like King, Pendorf and Chandrasekhar, but the main form has stayed the same and is still used today [2].

As shown in the section on atmospheric composition the majority of (gas)molecules in the atmosphere - specifically the dipoles N_2 and O_2 - are indeed much smaller than the wavelengths of visible light. This means that Rayleigh scattering occurs at every point in a clear atmosphere; diverting energy toward a viewer from all directions. Since the total scattering is a function of $1/\lambda^4$ the scattering is

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[2] Although this theory produces reasonably accurate results, one should note it still is only an approximation. Experiments do not yield exact $1/\lambda^4$ power results; Penndorf[2] e.g. obtained a inverse 4.089th-power scattering

greater at shorter wavelengths, or the violet/blue end of the visual spectrum. This effect had been observed earlier by John Tyndall[3] in 1859, who examined the passing of light through a clear fluid holding small particles in suspension.

In his original articles Rayleigh himself was unsure which particles could be responsible for this scattering and settled for salt. In 1889 he however published another article[1] in which he correctly argued that air molecules alone scatter sufficient light to be observable[4]. Particles of the same size *can* also scatter, but they are not needed. What *is* needed is that there is a dark background (the black universe) to see the scattering, and that the atmosphere is optically thin. Would earths atmosphere be much thicker, but identical in composition, the color would be different. This is illustrated by the fact that sunsets are red. If the sun is very close or below the horizon the light at grazing incidence must travel through a greater thickness of air then when overhead. More of the blue light is than scattered out of them beam, but cannot reach you due to the low position of the sun. Therefor, there will be relatively more red light present in the beams that *do* reach you.

2.3.2 Critical Opalescence

An alternative to Rayleighs theory on scattering was developed by Einstein in his 1910 article on Critical Opalesence, based on the work of Smoluchowski. Critical opalescence is the strong scattering that takes place in a system where the liquid and gas phase have the same density and liquid drops of the same size as the wavelength of visual light are formed. Einstein expanded on this to derive an equation for scattering without *directly* assuming that matter in the atmosphere is discreetly distributed. Instead matter he took matter to be continuous, but characterized by a refractive index that is a function of the position. The equation he derived for an homogenous ideal gas is:

$$\frac{J_0}{J_e} = \frac{RT_0}{N} \frac{(\epsilon - 1)^2}{p} \left(\frac{2\pi}{\lambda}\right)^4 \frac{\theta}{(4\pi D)^2} \cos^2 \phi \tag{4}$$

Which again shows the $1/\lambda^4$ wavelength dependency and therefor favors the scattering of violet over red light.

2.3.3 Mie Scattering

A more general theory than Rayleigh scattering is the Mie (or Mie-Debye) theory. It was first published by Gustav Mie in 1908 and describes the scattering of electromagnetic radiation by *spherical* and *homogenous* particles. It does so by calculating both the scattered *internal* field (all points inside the particle) as well as the scattered field at all points in the homogeneous material in which the particle is imbedded. Both the formulas and the calculations with it are far more complex than those of Rayleigh scattering (which is in fact a limit case of Mie theory for particles much smaller than the wavelength), and outside the scope of this article.

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[3] There is debate amongst physicists if this scattering effect should be named after Tyndall who observed it first (Tyndall effect) or Lord Rayleigh who derived the formula (Rayleigh Scattering). The majority seems to have chosen for the latter.

[4] a copy the article in question is attached for you reading pleasure

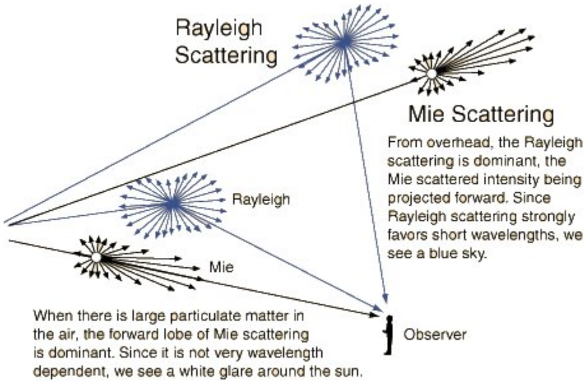

Figure 5: Rayleigh scattering vs Mie scattering

When looking at atmospheric scattering however it is necessary to mention it. In the regime where particles are about the size of the wavelength, like fog, haze, and pollution (aerosols) the scattering no longer conforms to the Rayleigh law. This scattering is called Mie-scattering and occurs mostly in the lower portions of the atmosphere where larger particles are more abundant - and dominates when cloud conditions are overcast. Contrary to Rayleigh scattering, Mie scattering favors a direction to scatter in: it is predominantly forward, diverting little energy to space. Another difference is that its scattering efficiency is not strongly dependant on the wavelength, meaning Mie scattered light appears almost white (hence the color of clouds and fog).

3 Color perception by the human eye

In the section on scattering it was shown that light with lower wavelengths is scattered more than light of higher wavelengths. Since it was also mentioned in the section on the light of the sun that the ”visual spectrum” -or the regime of wavelengths the human eye can see - ranges from *violet* to red the question why the sky is perceived blue instead of the even more scattered violet becomes apparent. To answer this question we must leave the realm of physics, and enter that of biology to explain how humans can see colors.

3.1 Cones and response curves

To perceive colors the human eye has three types of light receptors, known as cones, located in the retina. These cones only operate when there is enough illumination; at low light levels another type of detector called the rods take over. These rods however are colorblind; and as such not relevant for the perceiving of the color of the sky.

Each cone is sensitive to a broad range in wavelength, but each type has a different peak sensitivity. These peaks are found at wavelengths that we call red (580 nm), green (540 nm) and blue (450 nm). The perception of color comes from the brain’s interpretation of how many photons of particular wavelengths

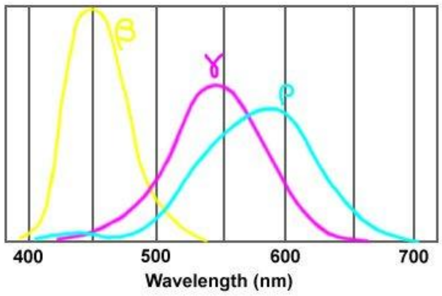

Figure 6: Response curves of the human eye. ρ denotes red, γ green and β blue.

fall within the response curves of each of the 3 types of cones. As can been seen in figure 6, the sensitives overlap. This means that if light of any color is intense enough it will be seen as white, since all cones will be fully activated. It also means there is more than one way to let the human eye perceive the same color; there is not a unique combination of wavelengths that e.g. creates lightgreen.

When looking at the sky, the red cones respond primarily to the small amounts of red light scattered, and less to the orange and yellow wavelengths. The green cones have their strongest response at the scattered green and green-blue wavelengths, and slightly less for yellow. Finally, colors near the strongly scattered blue wavelengths primarily stimulate the blue receptors. From this it can be concluded that the skylight stimulates the red and green cones almost equally, while stimulating the blue cones more strongly. By looking at the blue curve it can be seen that the sensitivity for blue light is much higher than that for violet. This means that even though there are more scattered ”violet wavelengths” than there are blue wavelengths, the blue is perceived better which results in humans perceiving the sky to be blue.

4 Conclusion

The sky appears to be blue on a clear day because the nitrogen and oxygen molecules in the atmosphere scatter the light of the sun. This scatteringproces, called Rayleigh scattering, scatters light of lower wavelengths better than that of higher wavelengths, approximately by $1/\lambda^4$. Due to the low optical thickness of Earths atmosphere it can be treated as an ideal gas so summation of the separate scatteres is allowed. The low optical thickness also ensures that most of the scattered light can actually reach the observer without losing too much intensity. Out of the visual spectrum violet and blue are scattered the most. The human visual system is more sensitive to blue than it is to violet, so perceives the sky to be blue.

References

[1] Rayleigh, Philos. Mag. 47, 375 (1899)

[2] Penndorf, J.Opt.Soc.Am. 47, 176182 (1957)

[3] Einstein, Ann.Phys. 33, 1275 (1910)

[4] Mie, Ann. Phys. 25, 377 (1908)

[5] http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

A Derivation of Rayleighs law

Consider a small homogeneous particle (like a molecule) with size smaller than the wavelength of incident radiation $\vec{E}_0$. Let $\vec{p}_0$ be the induced dipole moment and α the polarizabilty of the particle and classic electromagnetic theory gives:

$$\vec{p}_0 = \alpha \vec{E}_0 \tag{5}$$

The scattered electric field at the large distance r (called far field scattering) from the dipole is given (in cgs units) by

$$\vec{E} = \frac{1}{rc^2} \frac{\partial \vec{p}}{\partial t} \sin(\gamma) \tag{6}$$

where γ is the angle between the scattered dipole moment $\vec{p}$ and the direction of observation. In oscillating periodic field, the dipole moment is given in terms of induced dipole moment by

$$\vec{p} = \vec{p}_0 \exp(-ik(r - ct)) \tag{7}$$

and thus the electrical field is

$$\vec{E} = -\vec{E}_0 \frac{\exp(-ik(r - ct))}{r} k^2 \alpha \sin(\gamma) \tag{8}$$

Decomposing the electrical vector on two orthogonal components perpendicular and parallel to the plane of scattering (a plane containing the incident and scattering beams),we have

$$E_r = -E_{0r} \frac{\exp(-ik(r - ct))}{r} k^2 \alpha \sin(\gamma_1) \tag{9}$$

$$E_l = -E_{0l} \frac{\exp(-ik(r - ct))}{r} k^2 \alpha \sin(\gamma_2) \tag{10}$$

Using that

$$I = \frac{1}{\Delta\Omega} \frac{c}{4\pi} \|E\|^2 \tag{11}$$

the perpendicular and parallel intensities (or linear polarized intensities) are

$$I_r = I_{0r} k^4 \alpha^2 / r^2 \tag{12}$$

$$I_l = I_{0l} k^4 \alpha^2 cos^2(\theta) / r^2 \tag{13}$$

Using that natural light (incident beam) in not polarized($I_{0r} = I_{0l} = I_0/2$) and that $k = 2\pi/\lambda$, we have

$$I = I_r + I_l = \frac{I_0}{r^2} \alpha^2 \left(\frac{2\pi}{\lambda}\right)^4 \frac{1 + cos^2(\theta)}{2} \tag{14}$$

The Rayleigh scattering phase function for incident unpolarized radiation is

$$P(\cos(\theta)) = \frac{3}{4}(1 + cos^2(\theta)) \tag{15}$$

Equation 14 can be rewritten in the form:

$$I(cos(\theta)) = \frac{I_0}{r^2}\alpha^2\frac{128\pi^5}{3\lambda^4}\frac{P(\theta)}{4\pi} \tag{16}$$

Hence the scattering cross section σ_s (in units of area) by a single molecule is

$$\sigma_s = \alpha^2\frac{128\pi^5}{3\lambda^4} \tag{17}$$

The polarizability is given by the Lorentz-Lorenz formula:

$$\alpha = \frac{3}{4\pi N_s}\left(\frac{m^2-1}{M2+2}\right) \tag{18}$$

Where N_s is the number of molecules per unit volume and $m = n - ik$ in the refractive index